Chapter 1

The Incredible Bulk

Fundamental physics, as far as we have been able to probe experimentally, is described by two separate theories. The strong and electroweak interactions governing short distance are captured by the standard model of particle physics, while gravity, which dominates at long distance, is described by general relativity [1, 2]. The first is quantum, while the latter is classical, but both are locally Lorentz invariant field theories. We don't expect that either is truly fundamental, but when we say that about the standard model in the context of the LHC, it is in the mindset of effective field theory: we expect that it is the low energy limit of a more fundamental field theory. Reasoning similarly one might have thought that general relativity, which as a quantum field theory is expected to break down at the Planck scale, needs to be completed there by a fundamental quantum field theory of gravity. The truth is much more dramatic.

In locally Lorentz invariant field theories no signals propagate faster than light, and therefore no measurement can influence another measurement outside its future lightcone. As such, the field degrees of freedom at different points in space are independent and at any finite energy the number of accessible degrees of freedom, or entropy, scales as the volume. A series of developments over the last forty years have made a convincing case that this cannot be true in quantum gravity, and that therefore this framework has to break down.

In the seventies it was discovered that a black hole is a thermodynamic system, radiating as a black body at a temperature that vanishes in the classical limit $\hbar \to 0$, and with an entropy proportional to its horizon area [3, 4, 5]. If the second law of thermodynamics is to hold, this means the entropy of any spacetime region is bounded by the area enclosing it, because we can add matter to any system until a black hole forms, and the entropy cannot decrease in this process. Unless one is willing to give up quantum mechanics [6], i.e. time evolution as a unitary linear operator on a Hilbert space, the thermodynamic entropy must relate to the Hilbert space dimension of an underlying microscopic quantum theory. Attempts to modify quantum mechanics have all lead to truly unfixable problems such as a macroscopic breakdown of causality [7, 8], so one should not be willing to do this. The bizarre but only logical conclusion was drawn in the early nineties and is known as the *holographic principle* [8], stating that the number of degrees of freedom in any volume is equal to its bounding surface in Planck units, and that a description should exist in terms of degrees of freedom living on this surface.

If the holographic principle is correct, the framework of local quantum field theory has to break down at energy densities large enough to form a black hole, and at the shortest distances, spacetime and matter must be described by a quantum theory of something other than fields. It is plausible that this theory will still be Lorentz invariant, because while rotation invariant theories emerge naturally from microscopic theories without this symmetry through the renormalization group, this is not true for Lorentz invariant theories [9, 10].

String theory precisely fits the stated requirements. It is an exactly Lorentz invariant quantum theory of strings, not fields. It was also suspected that string theory is holographic soon after the formulation of the holographic principle [11], but this could not be proven without a non-perturbative formulation of string theory. A huge breakthrough was made in nineteen ninety seven, when it was conjectured that string theory on $d+1$ dimensional Anti-de Sitter space is equivalent to conformal quantum field theory on the d dimensional asymptotic boundary of that space [12, 13]. In the limit of large string tension and coupling the bulk theory reduces to local field theory that includes general relativity as a sector. The mapping between the bulk and boundary theories is complicated and so far the *AdS/CFT correspondence* has withstood proof, but there is an enormous amount of evidence for it [14]. Many generalizations have been found and since these don't necessarily involve AdS or CFT, but always relate a gauge theory to a theory of gravity in one higher dimension, they are referred to as *gauge/gravity dualities*.

The AdS/CFT correspondence provides an explicit model of holographic quantum gravity, where the non-gravitational boundary field theory is the holographic description of the gravitational *bulk* theory, and we can use it to investigate how the holographic principle actually works. It is incredible that the bulk theory is dual to a lower dimensional CFT. In the limit where the bulk theory becomes local this means that excitations that are coincident in the boundary CFT may be far apart in the bulk. From the point of view of the CFT they should be able to interact directly, but the bulk picture makes it clear that they cannot. The question of how bulk locality emerges from the boundary theory is therefore at the heart of holography. Although much research had been done in gauge/gravity duality before the work in this dissertation, none of it really answered this question [15].

Our universe is not AdS. It has a positive, rather than a negative cosmological constant, and will therefore look like de Sitter when it becomes dominated by the cosmological constant. dS does not have a timelike boundary on which a dual field theory naturally lives and formulating a holographic dual to it presents many challenges. The holographic principle applies to any region of spacetime, and ultimately the goal is to understand it for an arbitrary spacetime region, but for now AdS/CFT is all we have. When we understand it well enough it will hopefully be clear how to go beyond it. Since all AdS/CFT dualities share the same ten-dimensional flat space limit, the asymptotic structure of spacetime and many details of the boundary theory should play no role in the emergence of locality. We can therefore hope to extract the essential features of holography and learn lessons that remain valid with more realistic boundary conditions.

In this dissertation we describe three perspectives on the problem of bulk locality in gauge/gravity duality. In all cases we work in the limit where the bulk theory is a local field theory and attempt to explain this locality from the boundary perspective. In retrospect, the three perspectives correspond to the different ways of thinking about quantum field theory in the bulk: the S-matrix, the path integral and the canonical framework.

In chapter 2 we briefly review aspects of Anti-de Sitter spacetime, conformal and large N field theory, and gauge/gravity duality to supply the basics the reader will need in order to comprehend the later chapters and in the process establish notation.

In chapter 3 we conjecture that any conformal field theory (CFT) with a large-N expansion and a large gap in the spectrum of anomalous dimensions has a local bulk dual. We discuss the use of scattering experiments to probe the locality of bulk physics, and relate it to the form of the CFT four-point function. For an abstract CFT we formulate the consistency conditions for the boundary four point correlator, most notably crossing symmetry, and show that the conjecture is true in a broad range of CFTs, to first nontrivial order in $1/N^2$: in any CFT with a gap and a large-N expansion, the four-point correlator is generated via the AdS/CFT dictionary from a local bulk interaction. We establish this result from a counting argument on each side.

The key role played in the emergence of local bulk physics by a hierarchy in operator dimensions as well as the connection between bulk radius and boundary energy, suggest that the natural language in which to study bulk locality is that of the renormalization group. In chapter 4 we develop parallels between the holographic renormalization group in the bulk and the Wilsonian renormalization group in the dual field theory. Our philosophy differs from most previous work on the holographic RG; the most notable feature is the key role of multi-trace operators. We work out the forms of various single- and double-trace flows. The key question, 'what cutoff on the field theory corresponds to a radial cutoff in the bulk?' is left unanswered, but by sharpening the analogy between the two sides we identify possible directions.

In chapter 5 we discuss the construction of bulk local operators as smeared boundary operators. On the conceptual side, we use this to address the question

"If Schrodinger's cat were behind the horizon of an AdS black hole, could we determine its state by a measurement in the dual CFT?" and formulate conditions for uniquely identifying a set of CFT operators as the canonical bulk fields without already knowing the bulk dynamics. On the technical side, we extend the work of Kabat et al on the scalar field and correct their derivation of a spacelike scalar Green's function. Furthermore, using mode expansions, we obtain leading order smearing functions for gauge fields and gravity and we discuss to what extent a gauge field and the metric can be thought of as local observables.

Chapter 6 is a conclusion and discussion. We compare the different approaches and summarize what has been learned. Various technical results are collected in the appendix.

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